The Single-Sample $t$ Test

NOTE TO INSTRUCTORS

This chapter introduces the essential mastering of the most commonly used hypothesis test covered so far in the textbook—the $t$ test. Students who continue in this course of study will encounter $t$ tests in their future classes and in the psychology research articles they will read. Make sure that enough time and effort is spent in explaining the single-sample $t$ test as it will serve as the foundation for students to understand the chapters that will follow.

OUTLINE OF RESOURCES

I. The $t$ Distributions
   - Discussion Question 9-1 (p. 84)
   - Discussion Question 9-2 (p. 84)

II. The Single-Sample $t$ Test
    - Discussion Question 9-3 (p. 84)
    - Discussion Question 9-4 (p. 85)

III. Next Steps: Dot Plots
    - Discussion Question 9-5 (p. 86)
    - Additional Readings (p. 86)
    - Online Resources (p. 86)

CHAPTER GUIDE

I. The $t$ Distributions
   1. There are many $t$ distributions—one for each possible sample size. As the sample size becomes smaller, we are less certain that we are accurate and the $t$ distribution gets wider.
   2. The formula for standard deviation as estimated from a sample is:

$$s = \sqrt{\frac{\Sigma (X - M)^2}{(N - 1)}}.$$
3. This differs from the standard deviation formula that we used with samples where the denominator was only \( N \). By using \( N - 1 \), we get a slightly larger standard deviation in recognition that we are dealing with smaller samples and more error leading to greater spread in the distribution.

> Discussion Question 9-1

*Why do you use \( N - 1 \) rather than just \( N \) when calculating the standard deviation as estimated from a sample?*

Your students’ answers should include:
- Dividing by a slightly smaller number (\( N - 1 \)) increases the value of the standard deviation.
- A larger standard deviation helps to compensate for having smaller sample sizes and potentially more error.

4. We also refer to the standard deviation as \( s \) rather than \( SD \) to reflect that it is a corrected standard deviation, calculated to eliminate error.

5. The standard error as estimated from a sample is: \( s_m = \frac{s}{\sqrt{N}} \)

6. The *t* statistic indicates the distance of a sample mean from a population mean in terms of the standard error.

7. Calculating the *t* statistic is the same as calculating the *z* statistic except that the *t* statistic uses the estimated standard error rather than the actual standard error of the population mean. The formula for the *t* statistic is:

\[
 t = \frac{(M - \mu_M)}{s_M}
\]

> Discussion Question 9-2

*How does the *t* statistic differ from the *z* statistic?*

Your students’ answers should include:
- The *t* statistic uses the estimated standard error of the population of means; the *z* statistic uses the actual standard error of the population of means. The *t* statistic indicates the distance of a sample mean from a population mean in terms of the standard error.

II. The Single-Sample *t* Test

1. A *single-sample t test* is a hypothesis test in which we compare data from one sample to a population of which we know the mean but not the standard deviation.

> Discussion Question 9-3

*When would you use a single-sample *t* test?*

Your students’ answers should include:
- Use the single-sample *t* test when comparing data from a sample to a population for which you know the mean but not the standard deviation.
Because you only need to know the population mean, it has more uses than the \( z \) test, although its use is still fairly limited.

2. When we use the \( t \) table, we do not look up sample size. Instead, we look up the degrees of freedom (\( df \)), which are the number of scores that are free to vary when estimating a population parameter from a sample. With a single-sample \( t \) test, the formula for the degrees of freedom is: \( df = N - 1 \).

**Discussion Question 9-4**

*What are degrees of freedom? Why were they not necessary when discussing the \( z \) test?*

Your students' answers should include:

- Degrees of freedom are the number of scores that are free to vary when estimating a population parameter from a sample.
- The degrees of freedom allow us to take into account that we will have more confidence with larger samples and less confidence with smaller samples. There are different distributions for each possible sample size. However, for a \( z \) test, we are working with the entire population, not samples. As a result, there are no multiple \( z \) distributions, no degrees of freedom to account for different sample sizes, etc.

3. When conducting a single-sample \( t \) test, we use the same six steps of hypothesis testing. That is, we first identify the populations, distributions, and assumptions. Then, we state our null and research hypotheses. After that, our third step is to determine the characteristics of the comparison distribution. Fourth, we determine our critical values, or cutoffs. Fifth, we calculate the test statistic, and sixth, make a decision.

4. When writing the results of our hypothesis test, we need to write the symbol for the test statistic (e.g., \( z \) or \( t \)). Then, we write the degrees of freedom in parentheses. After that, our next step is to write an equals sign and then the value of the test statistic (usually to two decimal places). Lastly, we write a comma, then we decide whether the \( p \) value associated with the test statistic was less than or greater than the cutoff \( p \) level, and then include a 0 before the decimal point.

5. It is recommended that when we report the results of any hypothesis test, we also report the confidence intervals and effects sizes.

6. For the confidence interval, we draw a picture of the \( t \) distribution that includes the confidence interval. We then indicate the bounds of the confidence interval on the drawing, write the appropriate percentages under the segments of the curve, and look up the \( t \) statistics that fall at each line. Next, we convert the \( t \) statistics back into raw means and use the formula: \( M_{\text{lower}} = -t(\sigma) + M_{\text{sample}} \) and \( M_{\text{upper}} = t(\sigma) + M_{\text{sample}} \). Lastly, check that the confidence interval makes sense.

7. For effect size, we calculate Cohen’s \( d \) for a single-sample \( t \) test by using the formula: Cohen’s \( d = (M - \mu)/s \).
III. Next Steps: Dot Plots

1. The dot plot is a graph that displays all the data points in a sample, with the range of scores along the $x$-axis and a dot for each data point above the appropriate value.

2. Similar to the stem-and-leaf plots, a dot plot allows us to view the overall shape of the sample and to retain all the individual data points, but it also allows us to make comparisons between two groups a little easier.

3. To create a dot plot, we determine the range of scores, label the $x$-axis, include the values from lowest through highest, and then place a dot above the appropriate value for every score.

> Discussion Question 9-5

*Why would we want to use a dot plot? How is it useful?*
Your students’ answers may include:

- Use a dot plot to view the overall shape of the sample while retaining individual data points.
- Dot plots are useful in making comparisons between groups easier.

Additional Readings


This paper compares the properties of $t$ and $z$ in simulation runs. The differences in the results are attributed to the fluctuations in the $t$ test sample variance.


The $t$ test, like all parametric tests, has an underlying assumption that the data are normally distributed. What happens when our data are not normally distributed? Should we still use the $t$ test? Rasch and Volker argue yes.

Online Resources

This online psychology lab outlines numerous in-class experiments from which you can choose: http://opl.apa.org/Main.aspx. See their First Impressions experiment, which can be analyzed using a one-sample $t$ test.